Section - A (65 marks)

Q1). (i). (b) Null Matrix.

(ii). (b) $\frac{a^x . e^x}{\log_e(ae)} + C$

(iii). (d). $y^2 dx + (x^2 - xy - y^2)\, dy = 0$

(iv). (c) Both statements are true.

(v). (d) k only

(vi). (d). Assertion is false but reason is true.

(vii) (a). 1cm/min

(viii) (C). 2

(ix) (c). 4

(x). (a) $P(A) = P(B)$

(xi). (c) Both statements are true.

(xii). $R = \{(1,1)(2,2)(3,3)\}$.

(xiii). $x = 2$

(xiv) $\frac{60}{343}$

(xv) $5(2x+7)^{1/2} + C$

Q2). $\cos^{-1}(\sin(\cos^{-1}x)) = \frac{\pi}{6}$

$\sin(\cos^{-1}x) = \cos\frac{\pi}{6}$

$\sin(\cos^{-1}x) = \frac{\sqrt{3}}{2}$

$\cos^{-1}x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\cos^{-1}x = \frac{\pi}{3}$

$x = \cos\frac{\pi}{3}$

$x = \frac{1}{2}$

- Ans.

Q3).

$x^y \cdot y^x = 5$; T.P. $\frac{dy}{dx} = -\left(\frac{\log y + \frac{y}{x}}{\log x + \frac{x}{y}}\right)$

$\Rightarrow$ Taking log

$y \log x + x \log y = \log 5$

$\frac{y}{x} + \log x \frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$

$\left(\log x + \frac{x}{y}\right)\frac{dy}{dx} = -\left(\log y + \frac{y}{x}\right)$

$\frac{dy}{dx} = \frac{-\left(\log y + \frac{y}{x}\right)}{\left(\log x + \frac{x}{y}\right)}$

OR

$x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$

$\frac{dx}{d\theta} = a(1+\cos\theta)$; $\frac{dy}{d\theta} = a \sin\theta$

$\frac{dy}{dx} = \frac{a \sin\theta}{a(1+\cos\theta)}$

$= \frac{\cancel{a} \times \cancel{2} \sin\theta/2 \cancel{\cos\theta/2}}{\cancel{a} \times \cancel{2} \cos^2\theta/2} = \tan\theta/2.$

Q4). Let each side be 'x' cm.

$\frac{dx}{dt} = 2$ cm/sec.

$\left(\frac{dA}{dt}\right)_{x=20cm} = ?$

Clearly, $A = \frac{\sqrt{3}}{4} \times x^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2x \frac{dx}{dt}$

$\left(\frac{dA}{dt}\right)_{x=20} = \frac{\sqrt{3}}{\cancel{4}} \times 2 \times \overset{5}{\cancel{20}} \times 2$

$= 20\sqrt{3}$ cm²/sec.

$\therefore$ Area is increasing @ $20\sqrt{3}$ cm²/sec.

Q5).

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2}}$$

$$= \frac{1}{2}\log\left|x-\frac{9}{8}+\sqrt{x^2-\frac{9x}{4}}\right|+c$$

Q6).

$$\frac{dy}{dx} - e^x \times e^y = \frac{e^x}{e^y}$$

$$\frac{dy}{dx} = e^x\left(e^y+\frac{1}{e^y}\right)$$

$$\frac{dy}{dx} = e^x\left(\frac{e^{2y}+1}{e^y}\right)$$

$$\int \frac{e^y}{e^{2y}+1}\,dy = \int e^x\,dx$$

$$\tan^{-1}(e^y) = e^x + c$$

Q7).

$$\tan^{-1}\left(\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\frac{x^2-1}{x^2-4}}\right) = \frac{\pi}{4}$$

$$\frac{(x-1)(x+2)+(x+1)(x-2)}{x^2-4-x^2+1} = 1$$

$$\frac{x^2+\cancel{x}-2+x^2-\cancel{x}-2}{-3} = 1$$

$$2x^2-4 = -3$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$ - Ans.

Q8). $y = \sin^{-1}\left(\frac{2\times 6^x}{1+(6^x)^2}\right)$

$y = 2\tan^{-1}(6^x)$

$\frac{dy}{dx} = \frac{2}{1+36^x} \times 6^x \times \log 6$ — Ans.

Q9). $2y\, e^{x/y} dx + (y - 2xe^{x/y})dy = 0$

$2ye^{x/y} dx = (2xe^{x/y} - y)dy$

$\frac{dx}{dy} = \frac{(2xe^{x/y} - y)}{2y\, e^{x/y}}$

$\Rightarrow f(x,y) = \frac{2x\, e^{x/y} - y}{2y\, e^{x/y}}$

$\Rightarrow f(\lambda x, \lambda y) = \frac{2\lambda x\, e^{x/y} - \lambda y}{2\lambda y\, e^{x/y}}$

$= \frac{2xe^{x/y} - y}{2ye^{x/y}} = f(x,y)$

$\therefore$ it is homogeneous.

Also, $x = vy$; $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$v + y\frac{dv}{dy} = \frac{2vy\, e^{v} - y}{2ye^{v}}$

$v + y\frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}}$

$\cancel{v} + y\frac{dv}{dy} = \cancel{v} - \frac{1}{2e^{v}}$

$2\int e^{v} dv = -\int \frac{1}{y} dy$

$2e^{v} = -\log y + C$

$$2e^{x/y} = -\log y + c$$

Also, Putting $x=0$ & $y=1$

$$2 \times 1 = -0 + C \Rightarrow C = 2$$

$$\boxed{2e^{x/y} = -\log y + 2}$$

Q10). (a) $P(HHH) + P(TTT)$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{2}{8} = \frac{1}{4}$$

(b) $P(\text{diff. result}) = 1 - \frac{1}{4} = \frac{3}{4}$

(c). P(odd person in 4^{th} round)

$$= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{3}{256}$$

Q11). $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$; $X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$; $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$$

$$\Rightarrow x = 2;\ y = 3;\ z = 5$$

Q12). Per = 20 cm

$l + 2r = 20$

$r\theta + 2r = \cancel{20}\ 20$

$r(\theta+2) = 20$

(i) $r = \dfrac{20}{(\theta+2)}$

(ii) $A = \frac{1}{2} r^2 \theta$

or

$A = \frac{1}{2} l \times r$

$A = \frac{1}{2} r(20-2r)$

$A = (10r - r^2)$

$\dfrac{dA}{dr} = 10 - 2r$

$\dfrac{d^2A}{dr^2} = -2$

for max/min, $\dfrac{dA}{dr} = 0 \Rightarrow r = 5$

$\left(\dfrac{d^2A}{dr^2}\right)_{r=5} = -2 <$

∴ 'A' has max value when $r = 5$ cm.

Also, when $r = 5$

$\Rightarrow \cancel{5} = \dfrac{\cancel{20}\ 4}{\theta+2}$

$\Rightarrow \cancel{5}\ \theta + 2 = 4$

$\theta = 2^c$

(iii)

(iv). $A_{max} = 10 \times 5 - 25 = 25\ cm^2$

Q 13 (a) $\int_0^{\pi/2} \log\sin x = -\frac{\pi}{2}\log 2$

(b) $\int_0^{\pi/4} \log(1+\tan x) = \frac{\pi}{8}\log 2.$

Q14). $P(E_1)$ = Prob he knows the ans. $= \frac{1}{5}$

$P(E_2)$ = -- does not -- $= \frac{4}{5}$

$E \rightarrow$ he ans. it correctly

$P(E|E_1) = 1$

$P(E|E_2) = \frac{1}{5}$

$$P(E_1/E) = \frac{1 \times \frac{1}{5}}{1 \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{5+4}{25}} = \frac{5}{9}$$

Section B (15 marks)

Q15) (i). (b) $0 \leq \theta \leq \frac{\pi}{2}$

(ii). (d) $\frac{2\pi}{3}$

(iii) (a) $\frac{70}{11}$

(iv) $\frac{7}{\sqrt{17}}$ units

(v). $\frac{x+1}{4} = \frac{y}{4} = \frac{z-2}{4}$

$\Rightarrow x+1 = y = z-2$

Q16). Consider

$$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(15-12) + 2(-10+4) + 3(6-3)$$
$$= 3 - 12 + 9$$
$$= 12 - 12$$
$$= 0$$

$\therefore$ they are collinear.

Q17). let the eqn be

$$\boxed{a(x-2) + b(y-1) + c(z+1) = 0}$$

$(-1, 3, 4) \rightarrow a(-3) + b(2) + c(5) = 0$

$\Rightarrow -3a + 2b + 5c = 0$ — (1)

Also, $a - 2b + 4c = 0$ — (2)

solving (1) & (2)

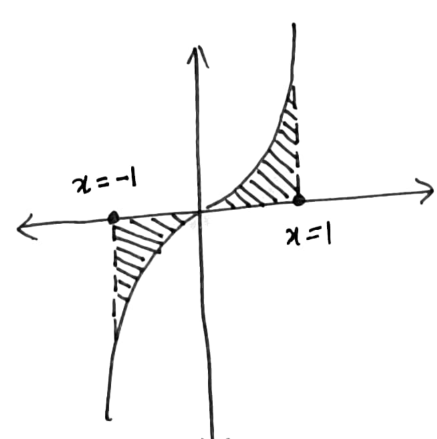
$a = 18k$; $b = 17k$; $c = 4k$.

eqn $18k(x-2) + 17k(y-1) + 4k(z+1) = 0$

$18x - 36 + 17y - 17 + 4z + 4 = 0$

$18x + 17y + 4z - 49 = 0$

Q 18). $y = x|x|$



Section C

19(i). (a)

(ii) (c)

(iii) $p = \frac{b}{(a+x)}$

$R(x) = \frac{bx}{(a+x)}$

$MR = \frac{ab}{(a+x)^2}$

$\frac{d}{dx}(MR) = \frac{-2ab}{(a+x)^3}$

Clearly $b<0$ & $a>$ $\Rightarrow$ $\frac{-2ab}{(a+x)^3} > 0$

$\therefore$ MR is increasing.

(iv). $C(x) = 35000 + 500x$

$R(x) = 5000x - 100x^2$

At, BEP, $R(x) = C(x)$

$35000 + 500x = 5000x - 100x^2$

$100x^2 - 4500x + 35000 = 0$

$x^2 - 45x + 350 = 0$

$(x-35)(x-10) = 0$

$x = 35$ or $x = 10$.

(v). $x + 2y - 5 = 0$ (y on x)

$2x + 3y - 8 = 0$ (x on y)

$\Rightarrow y = -\frac{x}{2} + \frac{5}{2} \Rightarrow b_{yx} = -\frac{1}{2}$

$x = -\frac{3}{2}y + 4 \Rightarrow b_{xy} = -\frac{3}{2}$

$r^2 = b_{yx} \times b_{xy} = \frac{3}{4}$

$r = -\frac{\sqrt{3}}{2}$

$\sigma_x = \sqrt{12}$

$b_{yx} = \frac{\sigma_y}{\sigma_x} \times r$

$+\frac{1}{\cancel{2}} = \frac{\sigma_y}{\sqrt{12}} \times +\frac{\sqrt{3}}{\cancel{2}}$

$\sigma_y = 2$

$\sigma_y^2 = 4$

20). $\Sigma(x_i-2) = 3 \Rightarrow \Sigma u = 3$

$\Sigma(y_i-7) = 2 \Rightarrow \Sigma v = 2$

$\Sigma x_i y_i = 83 \Rightarrow$

$\Sigma(x_i-2)(y_i-7) = -132 \Rightarrow \Sigma uv = -132$

$\Sigma(x_i-2)^2 = 55 \Rightarrow \Sigma u^2 = 55$

$\Sigma(y_i-7)^2 = 372 \Rightarrow \Sigma v^2 = 372$

$n = 7$

$$b_{yx} = \frac{\Sigma(uv) - \frac{\Sigma u \times \Sigma v}{n}}{\Sigma u^2 - \frac{(\Sigma u)^2}{n}}$$

$$b_{yx} = \frac{-132 - \frac{3\times 2}{7}}{55 - \frac{9}{7}} = \frac{-930}{376} = -2.47$$

$$\bar{x} = 2 + \frac{\Sigma u}{n} = 2 + \frac{3}{7} = \frac{17}{7}$$

$$\bar{y} = 7 + \frac{\Sigma v}{n} = 7 + \frac{2}{7} = \frac{51}{7}$$

eq^n of y on 'x'

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - \frac{51}{7} = -2.47\left(x - \frac{17}{7}\right)$$

$$\boxed{y = -2.47x + 13.28}$$

21). $R(x) = 3x^2 + 36x + 5$

(i) $AR = 3x + 36 + \frac{5}{x}$

(ii) $MR = 6x + 36$

(iii) $MR(x=5) = 30 + 36 = ₹66$

(iv) Revenue by selling 50^{th} item $= R(x=50) - R(x=49)$

$$= 3(50)^2 + 36(50) - 3(49)^2 - 36(49)$$
$$= 3 \times 99 + 36 = ₹333$$

OR

$$4 = 1400a + b$$
$$\underline{-\ 2 = 1800a + b}$$
$$2 = -400a \Rightarrow a = -\frac{1}{200}$$

$$4 = -7 + b \Rightarrow b = 11$$

$$\therefore\ p = -\frac{x}{200} + 11$$

$$R(x) = -\frac{x^2}{200} + 11x$$

$$AR = -\frac{x}{200} + 11$$

$$MR = -\frac{x}{100} + 11$$

When $MR = 0 \Rightarrow x = 1100$

$$\Rightarrow p = \frac{-1100}{200} + 11 = ₹5.50$$

22). $Z = -50x + 20y$ (to be minimised)

$2x - y \geq -5$

Consider

$2x - y = -5$

x	0	-2.5
y	5	0

Considering,

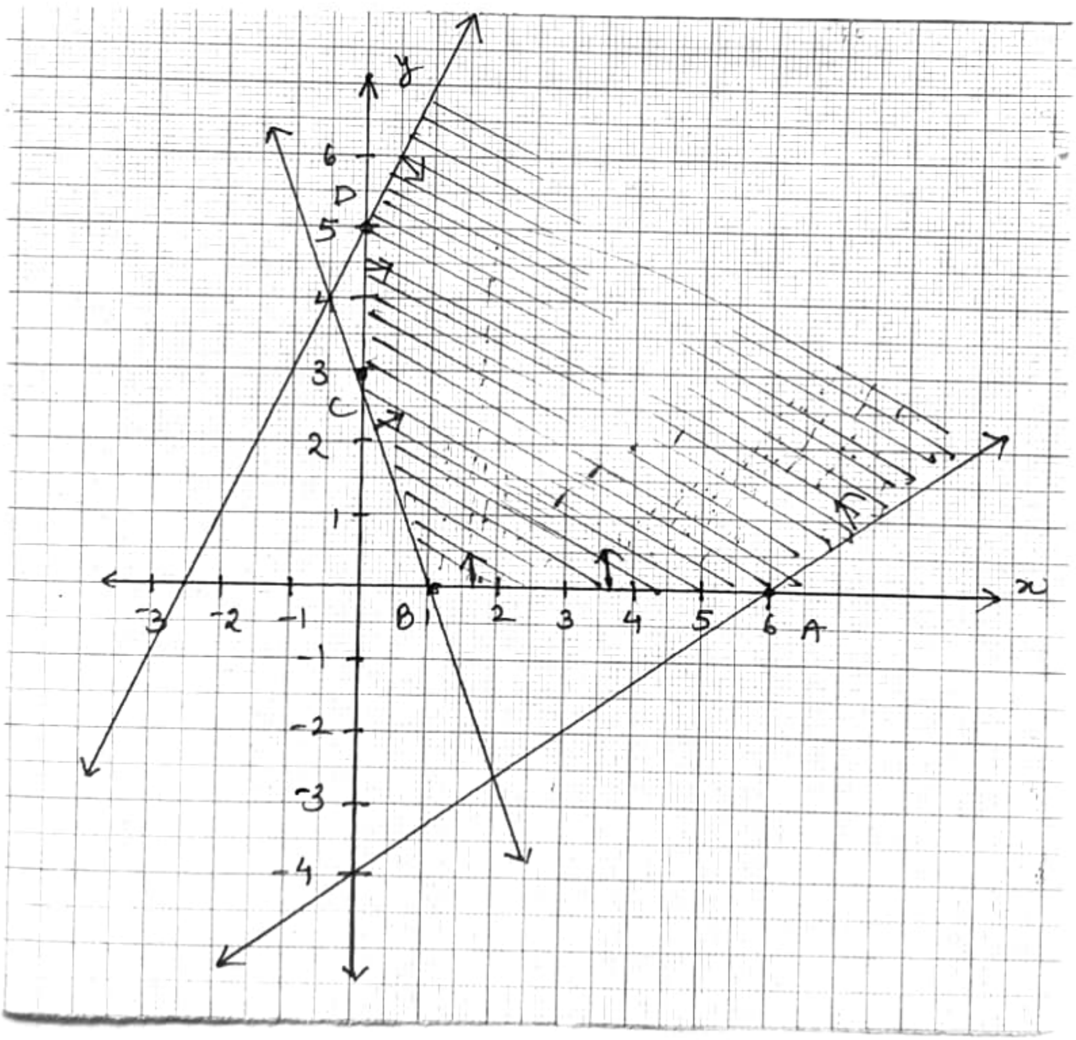
$3x + y \geq 3$

x	0	1
y	3	0

Consider

$2x - 3y \leq 12$

x	0	6
y	-4	0



A(6,0) ; B(1,0) ; C(0,3) ; D(0,5)

clearly, Min $Z = -300$