

General Instructions:

(Candidates are allowed additional 15 minutes for **only** reading the paper.

They must NOT start writing during this time.)

The Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from **Section A** and all questions EITHER from

Section B OR Section C

Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B: Internal Choice has been provided in one question of two marks each one question of four marks each.

Section C: Internal Choice has been provided in one question of two marks each one question of four marks each.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets.

Mathematical tables and graph papers are provided.

Section A (80 marks)

Question1)

[15X1]

In sub-parts (i) to (x) choose the correct options and in sub-parts (xi) to (xv), answer the questions as instructed.

(i) If $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, then A^{16} is

- (a) Unit Matrix (b) Null Matrix (c) Diagonal Matrix (d) Skew-Symmetric Matrix

(ii) The value of $\int a^x \cdot e^x dx$ equals

- (a) $(a^x \cdot \log_e a)e^x + C$ (b) $\frac{a^x \cdot e^x}{\log_e(ae)} + c$ (c) $\frac{a^x \cdot e^x}{\log_{ae}(e)} + c$ (d) $\log_e(ae)((ae)^x) + C$

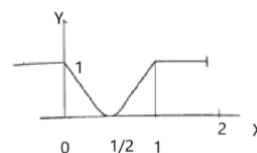
(iii) Which of the following is a homogeneous differential equation?

- (a) $(4x^2 + 6y + 5)dy - (3y^2 + 2x + 4)dx = 0$ (b) $(xy)dx - (x^3 + y^3)dy = 0$
 (c) $(x^3 + 2y^2)dx + 2xydy = 0$ (d) $y^2dx + (x^2 - xy - y^2)dy = 0$

(iv) Consider the graph of the function $f(x)$ shown along-side :

Statement 1 The function $f(x)$ is increasing in $(\frac{1}{2}, 1)$

Statement 2 The function $f(x)$ is strictly increasing in $(\frac{1}{2}, 1)$



Which of the following is correct with respect to the above statements?

- (a) Statement 1 is true and statement 2 is false.
 (b) Statement 2 is true and statement 1 is false.
 (c) Both the statements are true.
 (d) Both the statements are false.

(v) The existence of unique solution of the system of equations $x + y = \mu$ and $5x + ky = 2$ depends on

- (a) μ only (b) $\frac{\mu}{k} = 1$ (c) both k and μ (d) k only

(vi) **Assertion:** Degree of the differential equation: $a\left(\frac{dy}{dx}\right)^2 + b\left(\frac{dy}{dx}\right) = c$ is 3

Reason: If each term involving derivatives of differential equation is a polynomial

(or can be expressed as a polynomial) then highest exponent of the highest order derivative is called the degree of the differential equation.

Which of the following is correct?

- (a) Both Assertion and reason are true and reason is the correct explanation for assertion.
- (b) Both Assertion and reason are true and reason is not the correct explanation for assertion.
- (c) Assertion is true but reason is false.
- (d) Assertion is false but reason is true.

(vii) A cylindrical popcorn tub of radius 10 cm is being filled with popcorns at the rate of 314 cm^3 per minute.

The level of the popcorns in the tub is increasing at the rate of:

- (a) 1cm/min (b) 0.1 cm/min (c) 1.1 cm/min (d) 0.5 cm/min

(viii) If $f(x) = \begin{cases} x+2, & x < 0 \\ -x^2 - 2, & 0 \leq x < 1, \\ x, & x \geq 1 \end{cases}$ then the number of points of discontinuity of $f(x)$, is/are

(a) 1 (b) 3 (c) 2 (d) 0

(ix) The sum of the degree and the order of the differential equation $\frac{d}{dx} \left[\left(\frac{d^2 y}{dx^2} \right)^5 \right] = 0$ is

- (a) 3 (b) 5 (c) 4 (d) none of the above

(x) If A and B are two events such that $P(B/A) = P(A/B)$, then

- (a) A is a subset of B (b) $A = B$ (c) $A \cap B = \emptyset$ (d) $P(A) = P(B)$

(xi) **Statement 1:** If A is an invertible matrix then $(A^2)^{-1} = (A^{-1})^2$

Statement 2: If A is an invertible matrix then $|A^{-1}| = |A|^{-1}$

- (a) Statement 1 is true and statement 2 is false.
- (b) Statement 2 is true and statement 1 is false.
- (c) Both the statements are true.
- (d) Both the statements are false.

(xii) Write the smallest equivalence relation from the set A to set A, where $A = \{1, 2, 3\}$

(xiii) For what value of x, is $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

(xiv) Three critics review a book. Odds in favour of the book are 5:2, 4:3 and 3:4 respectively for the three critics.

Find the probability that all critics are in favour of the book.

(xv) Evaluate: $\int \frac{5}{\sqrt{2x+7}} dx$

Question2)

Solve for x, $\cos^{-1}(\sin(\cos^{-1} x)) = \frac{\pi}{6}$ [2]

Question3)

If $x^y y^x = 5$, show that $\frac{dy}{dx} = -\left(\frac{\log y + \frac{y}{x}}{\log x + \frac{x}{y}}\right)$ [2]

OR

If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, then find $\frac{dy}{dx}$

Question4)

The side of an equilateral triangle is increasing at the rate of 2cm/s. At what rate is its area increasing, [2]

when the side of the triangle is 20 cm?

Question5)

[2]

Evaluate: $\int \frac{1}{\sqrt{4x^2-9x}} dx$

OR

Evaluate: $\int \frac{1}{x^2-x^3} dx$

Question6)

[2]

Solve the D.E.: $\frac{dy}{dx} - e^{x+y} = e^{x-y}$

Question7)

Solve: $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$

[4]

Question8)

[4]

Differentiate: $\sin^{-1} \frac{2^{x+1} 3^x}{1+36^x}$

Question9)

[4]

Show that the differential equation $2ye^{x/y} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$ is homogeneous.

Find the particular solution of the differential equation, given that $x = 0$ when $y = 1$.

Question10)

[4]

Three friends go to a restaurant to have Pizza. They decide who will pay for the pizza by tossing a coin. It is decided that each one of them will toss a coin and if one person gets a different result than the other two, that person would pay. If all three get the same result, they will toss again until they get a different result.

- What is the probability that all three friends will get the same result in one round of tossing?
- What is the probability that they will get different result in one round of tossing?
- What is the probability that they will need exactly four rounds of tossing to determine who would pay?

Question11) Use Matrix method to solve the following system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \text{ \& } \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

[6]

OR

Show that if $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$

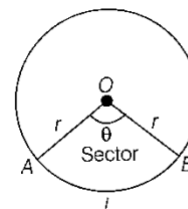
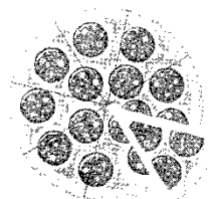
Question12) The length of the perimeter of a slice of a pizza in the form of a sector of circle is 20 cm.

r be the radius of the circle, sectorial angle be θ and l be the length of the arc.

Based on the above in the information, answer the following questions.

- Express the radius of the sector is expressed in terms of sectorial angle be θ radian
- Let A be the area of the slice. Then, express A in terms of r .
- For the maximum value of A , find the value of the sectorial angle.
- Maximum area of the slice of the pizza is.

[6]



Question13)

[6]

(a) Evaluate: $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$.

(b) Evaluate: $\int_0^{\frac{\pi}{4}} \log (1 + \tan x) dx$

Question 14)

A student answers a multiple-choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is $\frac{1}{5}$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, find the probability that he did not tick the answer randomly.

[6]

Section B (15 marks)

Question 15)

[5 X1]

In sub-parts (i) to (iii) choose the correct options and in sub-parts (iv) and (v), answer the questions as instructed.

(i) If θ is the angle between two vectors \vec{a} and \vec{b} then $\vec{a} \cdot \vec{b} \geq 0$, when

- (a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$ (c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

(ii) If \vec{a} and \vec{b} are unit vectors and θ is the angle between them, then $\vec{a} + \vec{b}$ is a unit vector, if

- (a) $\theta = \frac{\pi}{4}$ (b) $\theta = \frac{\pi}{6}$ (c) $\theta = \frac{\pi}{2}$ (d) $\theta = \frac{2\pi}{3}$

(iii) The value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular

- (a) $\frac{70}{11}$ (b) $\frac{11}{70}$ (c) $-\frac{70}{11}$ (d) $-\frac{11}{70}$

(iv) Find the distance of the point $(0, -3, 4)$ from the plane $3x + 2y + 2z + 5 = 0$.

(v) Find the Cartesian equation of the line passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$.

Question 16)

[2]

Check whether $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{a} - 3\vec{b} + 5\vec{c}$ are coplanar.

OR

Show that: $\vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = [\vec{a} \vec{b} \vec{c}]$

Question 17)

[4]

Find the equation of the plane through the points $(2, 1, -1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$

OR

Find the Cartesian equation of the plane passing through the intersection of the planes

$\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $3x - y + 4z = 0$ and at a unit distance from the origin.

Question 18)

[4]

Sketch the region enclosed bounded by the curve, $y = x|x|$ and the coordinates $x = -1$ and $x = 1$.

Section C (15 marks)

19. In sub-parts (i) and (ii) choose the correct option and in sub-parts (iii) to (v), answer the questions as Instructed. [1X5]

(i) A firm has the cost Function $C = 2x^3 + \frac{x^2}{2} - 5$ and demand function $x = 100 - p$, the Profit Function is

- (a) $100x - \frac{3}{2}x^2 - 2x^3 + 5$ (b) $\frac{x^3}{3} - 5x^2 - 50x + 5$ (c) $\frac{x^3}{3} - 5x^2 - 50x - 5$ (d) None

(ii) If $\bar{x} = 15, \bar{y} = 5, \sigma_x = 12, \sigma_y = 2.4$ and $r = 0.8$, then regression line of x on y will be

- (a) $x + 4y - 5 = 0$ (b) $x - 4y - 10 = 0$ (c) $x - 4y + 5 = 0$ (d) None

(iii) For the demand function $= \frac{b}{a+x}$, show that the marginal revenue function

is increasing for all $b < 0, a > 0$.

(iv) The fixed cost of new product is Rs. 35000 and the variable cost per unit is Rs. 500. If the demand function is $p(x) = 5000 - 100x$, find the break even - value (s).

(v) The two lines of regression are $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and the variance of x is 12.

Find the variance of y and the coefficient of correlation.

20. For a bi-variate data, the following information is obtained:

$\sum(x_i - 2) = 3, \sum(y_i - 7) = 2, \sum x_i y_i = 83, \sum(x_i - 2)(y_i - 7) = -132, \sum(x_i - 2)^2 = 55, \sum(y_i - 7)^2 = 372, n = 7$.

Find the line of best fit for y.

[2]

OR

The data for marks in Physics and History obtained by 10 students are given below:

Marks in Physics	15	12	8	8	7	7	7	6	5	3
Marks in History	10	25	17	11	13	17	20	13	9	15

Using this data : Find the line of regression in which Physics is taken as a independent variable.

21. The total revenue received from the sale of x units of a product is given by

$$R(x) = 3x^2 + 36x + 5. \text{ Find}$$

[4]

- (i) the average revenue
(ii) the marginal revenue
(iii) the marginal average revenue when $x = 5$
(iv) the actual revenue from selling 50th item.

OR

A monopolist's demand function for one of its production is $p(x) = ax + b$. He knows that he can sell 1400 units when the price is Rs. 4 per unit and he can sell 1800 units at a price of Rs. 2 per unit.

Find the total, average and marginal revenue function. Also, find the price per unit when the marginal revenue is zero.

22. Solve the following linear programming problem graphically :

[4]

Maximize $Z = -50x + 20y$ subject to the constraints :

$$2x - y \geq -5; 3x + y \geq 3; 2x - 3y \leq 12; x \geq 0; y \geq 0$$